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ABSTRACT

Mathematics problem solving skills need to be developed through meaningful interactions. Computer simulations combined with cooperative learning methods provide students a context within which to construct an understanding of mathematics problem solving. This study examines the distribution and sequence of events that result in acquisition of those problem solving skills. Subject pairs of middle school age were observed solving mathematics problems presented by a computer simulated microworld. It appears that a complex set of strategies resulting in successful solution of the mathematics problems are developed by the subjects during the successful completion of the computer simulation and those strategies may be acquired within pairs of subjects. This study indicates that for this limited case mathematics problem solving skills along with cooperative communications can be a positive supplement to more traditional instruction. However, additional work in this area is indicated to assess the extent and magnitude of the learning taking place during the completion of the computer simulation. (2 figures, 43 references) (Author/BBM)

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Author:

Bernard W. Arenz

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Mathematics Problem Solving within a
Cooperative Learning Computer Structure

Bernard. W. Arenz
Assistant Professor

Southwest Missouri State University
Elementary and Secondary Education
901 South National Avenue
Springfield, MO 65807

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Mathematics problem solving skills need to be developed through meaningful interactions. Computer simulations combined with cooperative learning methods provide students a context within which to construct an understanding of mathematics problem solving. This study examines the distribution and sequence of events that results in acquisition of those mathematics problem solving skills. Subject pairs of middle school age were observed solving mathematics problems presented by a computer simulated microworld. It appears that a complex set of strategies resulting in successful solution of the mathematics problems are developed by the subjects during the successful completion of the computer simulation and that those strategies may be acquired within pairs of subjects.

Mathematics problem solving continues to be a topic of concern for mathematics educators. Mathematical problem solving requires a student of mathematics to possess knowledge about a mathematical domain and to apply that knowledge with understanding to new and unique situations. Early attempts to teach mathematical problem solving through direct instruction met with mixed results (Gagne, 1965). Alternate views hold that the transition from novice to expert requires a person-to-person interaction in which the knowledge and understanding that comprises expert behavior is created and shared during meaningful problem solving activities (Bruner, 1960). The true test of mathematical understanding is in the application of mathematical thinking to solve appropriate problems (Schoenfeld, 1988). The Curriculum and Evaluation Standards for School Mathematics (Standards) published by the National Council of Teachers of Mathematics in 1989 recommends that problem solving form the basis for mathematics instruction. The task of mathematics education, then appears to be one of providing students with opportunities to develop an understanding of mathematics problem solving that may be applied to a variety of problem solving situations.

Cooperative learning theory provides a methodology in keeping with the overall goal of developing mathematics problem solving skills consistent with the Standards. Cooperative groups provide a context that promotes and supports the person-to-person interaction needed for the development of problem solving knowledge and understanding. These groups, however, must engage in shared experiences and must be structured

in such a way that students are encouraged to cooperate with each other in order to accomplish a common goal. This common goal develops positive interdependence between the students so that the success of each is dependent upon or enhanced by the knowledge and understanding of each other (Johnson, 1975). Thus cooperative learning provides the instructional framework for such a transmission of knowledge and understanding between learners.

The context within which that transmission takes place, however, must also be considered. Mathematics problem solving is thought to be best practiced within the context of familiar real-world problems. Students need to feel a connection to the problems that are presented. Perceived relevance of the problem by the problem solver is related to how closely the problem is grounded in the reality of the problem solver. The Standards (1989, p. 66) recommend that, "in developing the problem situations, teachers should emphasize the application of mathematics to real-world problems as well as to other settings relevant to the middle school students." In the case of mathematics text and workbooks the problems appear all too often to be contrived and not particularly relevant to the personal needs of the students. For the purposes of this study, a computer simulation that presents problems within the appropriate realm of the characters in a simulation is used to provide a relevant context for problem solving.

In summary, mathematics problem solving skills need to be developed through cooperative learning environment which allows individual problem solving skills to be reified and shared between students. The computer provides the students a context within which they are able to construct an understanding of mathematics problem solving. This study examines the process of learning mathematics problem solving skills, and the way in which a cooperative learning environment provides the forum for the transmission of those skills, all within a computer assisted learning context.

Purpose

The purpose of this study is to examine the process of learning mathematics problem solving skills and the way in which a cooperative learning environment provides the forum for the transmission of those skills between students within a computer assisted learning structure. The following three areas were investigated: (1) The distribution of mathematics problem solving characteristics used by students in solving mathematics problems, (2) the distribution of cooperative communications skills used by students in solving mathematics problems and (3) the acquisition of mathematics problem solving characteristics and cooperative communications skills.

The first question addresses the characteristics of mathematics problem solving. Students must proceed with a problem solving task with a complex set of actions that will lead them toward a solution of that problem. The actions are based upon the current resources, heuristics, control structures and beliefs of the student at the time of the activity (Schoenfeld, 1985). These mathematics problem solving characteristics develop over a period of time and may or may not directly relate to the computer based problem at hand. It is important to be able to categorize the problem solving strategies used by students in a computer based problem solving activity so that an accurate model of learner behavior can be built.

The second question is important to the examination of the effects that cooperative communications may have on the problem solving task. Difference in communication may take place when students are given the opportunity to work with each other on a problem solving task. In a traditional competitively structured classroom where all students compete with each other for grades, there may not be sufficient incentive for students to help each other. When new concepts are presented in a computer problem solving mathematics activity that are not easily understood, students may not feel comfortable asking peers for assistance and conversely, peers may be reluctant to give assistance. In the cooperative condition, however, the expectation is just the opposite (Johnson and Johnson, 1975). All students are required to help each other learn and to not do so would be a violation of class requirements and officially sanctioned behavior. What then happens when students are working together on a computer based problem solving task that requires them to communicate with each other in order to solve the problem?

The third question examines the acquisition of skills and understanding associated with the problem solving task. Each student brings a set of characteristics, as defined by Schoenfeld, to the problem solving task. Johnson and Johnson describe cooperative learning as a way in which skills may be shared with and learned from each other. It would be helpful to identify a pattern of problems solving characteristics and cooperative communication skills that become common to both participants during the computer based problem solving simulation.

The answers to these three questions should be helpful in dealing with the appropriate use of the currently scarce computer resources if learning mathematical problem solving skills can be shown to be enhanced by pairing students on computers while using cooperative learning strategies.

Significance

The way in which mathematics problem solving proceeds and how that may be taught within a classroom has long been a topic of interest to educators. The early hypothesis that problem solving across all activities was universally generalizable has given way recently to a belief that problem solving may be more content specific. This has in turn regenerated interest in mathematics problem solving as an activity requiring more finely tuned investigation. Mathematics problem solving activities that take place in a computer environment need to be explored within the broader context of mathematical problem solving but also as a special subset of that investigation. A concerted effort by the educational research community has not as yet been mounted in this area.

Rather sweeping changes in mathematics education are underway. Every student must "become a mathematics problem solver" and also "learn to communicate mathematics (Standards, p. 6)." The Standards are being used as an impetus to reexamine the mathematics curriculum in schools across the country and make changes in the way that mathematics instruction takes place in elementary and secondary schools. These changes are also have an affect on teacher training as colleges prepare preservice and inservice teachers for a changing mathematics curriculum. It is important to develop an increased understanding of the proper role that computers can

play in assisting students to develop both mathematics problem solving and communication skills during this period of transition from a more traditional curriculum to one that mathematics educators move toward that is based upon the Standards.

The overall social environment within schools that technology threatens to fragment and isolate in the quest for higher achievement performance may be preserved through restructuring computer task structures. Cooperative learning techniques combined with judicious use of computers could prove to be a combination that provides the verifiable academic progress that is an expectation of schools at the same time as encouraging student to student interactions.

The increased interest to both give students computer access and also teach mathematics problem solving techniques has resulted in a number of computer programs being used for this combined purpose. With computers continuing to be a scarce resource, cooperative learning techniques may be an effective means of providing increased computer access at the same time as improved learning of problem solving skills.

The additional benefit may be that student-to-student social interaction goals may be advanced at the same time along with the other positive results of cooperative learning shown by other studies in this area. A demonstration of a cooperative computer learning environment improving performance would be valuable for educators primarily concerned with measurable performance. An environment that enables students to interact freely as they construct problem solving strategies may soften the use of technology for those critics of computers who are most concerned with the consequences of limited interpersonal interactions in computer assisted learning activities.

Literature

Problem solving activities in mathematics require skills and understandings that are often not readily apparent to the novice problem solver. It has been a goal of mathematics educators to provide students with the skills necessary for success in problem solving. Polya (1945) developed a set of problem solving steps that are claimed to be essential in problem solving and when properly applied to a mathematics problem: Step 1: understanding the problem, step 2: devising a plan, step 3: carrying out the plan; and step 4: looking back.

Schoenfeld extended Polya's work in order to develop a more complete description of the difference between a novice and expert mathematics problem solver. He studied the problem solving techniques of sophomore college students who had spent at least a year in the study of Euclidian geometry during high school. The subjects in this study were given typical problems in geometry and their attempts at solving the problems were videotaped. This investigation indicates that problem solving in this area may be characterized into four areas of: student knowledge and behavior; resources; heuristics; control and belief system.

In a cooperative learning situation these may be shared between the students so that the pooled resources may be applied to the task. The interactions of the students while engaged in cooperative computer problem solving exercises will be recorded.

The recorded interactions will then be examined for similarities and differences between this and problem solving interactions.

One area of investigation by Johnson, Johnson, Pierson and Lyons (1985) seems particularly intriguing, that of the use of controversy in goal setting. This research compared the use of controversy versus concurrence seeking as group goals. It was found that the groups with controversy as a goal showed higher achievement, motivation and a more accurate perspective than the groups with a goal of concurrence.

In the cooperative learning structure, the student and the instructor are brought together in a common bond based upon the humanistic needs inherent in teaching and learning. This bond is the manifestation of the love of learning expressed by Noddings. "The subject and student must be linked inseparably in the act of teaching. Linked to this passion for the material, but probably more important, is love of the acts of teaching and learning." (Noddings, 1984b, p. 157) This intrinsic connection between subject, teaching and learning continues as reflected in the relationships that both Noddings and the Johnsons propose as the basis for education.

"There is both an intellectual (or cognitive) representation and an affective one. If the activity is mainly intellectual, the intuition gives over its representation to reason, and objects are thought. If the activity is mainly moral, the intuitive representation is nearly pure feeling striking the motivational center with the force of its own being." (Noddings, 1984b, p. 73)

"I let the object act upon me, seize me, direct my fleeting thoughts as I scan the structures with which I may, in turn act upon the object. My decision to do this is mine; it requires an effort in preparation, but it also requires a letting go of my attempts to control. It involves a deliberate giving-over of subjectiveness." Noddings (1984b, p. 74) proposes that this giving over must be a conscious and deliberate act, "with a purposive quest for meaning and understanding."

Johnson & Johnson propose a curriculum that is guided by the need for cognitive, affective and content development needs of the learner. In this environment, each learner has the ability to grow and develop both socially and academically. This development is accomplished through activities that bind students together in cooperative groups. It is the group that forms the basis for both academic and social development. The distinguishing characteristic that separates the Johnsons' methodology from other group or team structures is the principle of positive interdependence. In positive interdependence, each student is responsible for the knowledge and welfare of the other group members. This is accomplished by setting external goal structures that require the common understanding of all group members. This could be contrasted with team work in which each member of the team is assigned a specialized task. Although the team needs each member to succeed in its primary function, team members are not responsible for acquiring the knowledge of other team members. In a learning environment, it is desirable that each team member is responsible for the knowledge and understanding of all other members. This will increase the helping behavior of all participants. Studies have shown that the gains in learning are highly correlated with explanation giving and receiving. Both the recipient and provider of information benefit from increased interaction that is focused on the knowledge and understanding necessary to complete the task at hand.

The emphasis on mutual responsibility for achievement sets up the cooperative relationship (Johnson, 1988). We all join together to realize the goals of education. In this way the teacher and students are dependent upon each other for their wellbeing. These elements combine to change both the motivation for learning and the way in which learning takes place. "The more cooperative a student's attitude, the more they see themselves as being intrinsically motivated." (Johnson, 1988, 3.12) The locus of control of the learning process is thus shifted from the external responsibility of the one-caring to the cared-for. The group process also provides an "...enduring effort aimed at achieving *satori*, or the sudden flash of enlightenment that comes after long, intensive, but successful effort." (3.15) These two elements combine to shift the focus of instruction from short term externally motivated skill acquisition to the pursuit of understanding and meaning.

Students must have the opportunity to work together so that they can reflect upon both the content and process used to acquire knowledge. In the competitive condition there is a negative incentive structure that limits or precludes the sharing of insight into the process. If the "secret" to success is shared, others will be more competitive thus lessening the chance for superior performance. An individualistic structure has unnatural avenues for the learner to share knowledge and understanding. Even with a strong internal tendency for the individual to assume a one-caring role the opportunity to fulfill that role may never present itself.

"Children and adolescents learn to view situations and problems from perspectives other than their own through their interaction with peers." The group work provides an opportunity for each member to take the one-caring role as well as the cared-for role. This shift allows students to look at themselves as others see them during the role shifts. (Johnson, 1988, 3.6)

Research in cooperative learning methods applied to computer assisted instruction activities indicate that computers may be used to further instructional goals that require group work. Students in the eighth grade were studied as they worked on a computer geography simulation in cooperative, individual and competitive modes of instruction (Johnson et al., 1986). The researchers concluded from this investigation that there were positive effects for the subjects in the cooperative groups during a computer assisted simulated environment.

Cooperative learning research in a computer environment is limited, but sufficient enough to both indicate promise and yet require additional inquiry. Microcomputer interactions, however, are similar to interactions within other learning activities. Research carried out on the problem solving practice with microcomputers (Light et. al., 1987) found that 11 year old children paired together in a structured interaction were more likely to solve complex problems correctly. These results, it was pointed out, were similar to results obtained by Glachan & Light (1982) who conducted similar research using physical objects instead of the microcomputer.

It is reasonable to expect that although microcomputer environments do differ from more traditional learning environments, the interactions of students with each other in cooperative groups may be very similar. Current research into cooperative learning supports cooperative learning as a means for increasing academic achievement and a more positive attitude towards school and other students. This study is intended to extend the investigations into the relationship between mathematics problem solving,

cooperative communications and a computer assisted instruction simulated microworld.

METHOD

This study incorporated group work, interaction characteristics, sequential analysis techniques and statistical results along with observational data. The interactions of pairs of students engaged in problem solving strategies and cooperative behaviors during the computer simulated mathematics problems were categorized and analyzed.

The instructional grouping in all cases was pairs of randomly selected students working together on the computer problem solving problem. Observations are broadly categorized as cooperative and problem solving. Sub categories for each of these characteristics are given in the section on observations. Analysis of the observation is divided into transitional probabilities and frequencies. Results are reported in the form of frequency graphs, tables and interaction matrices.

Treatment

Subjects worked on McGraw-Hill problem solving activities. This software creates an imaginary world in which the problems to solve are grounded. Subjects had the opportunity to solve problems that are both relevant to the characters in the story and lead to the solution of an overall problem. Preliminary investigation indicated that this simulated world is very motivating for the students. Problems in negotiating the simulated three dimensional space that the solution script occupies provided a varied experience that students found appealing.

Mathematics problem solving courseware published by McGraw-Hill and developed by Tom Snyder Productions was used as the basis of both learning activities. This series was built around a story line that had the student solving a mystery through a series of movements through a simulated building or geographic area. Each of the software packages were given grade level recommendations of 5 through 8. Each program started with a simple mathematics computation problem. The mathematics problems became more complex as the student(s) moved towards attainment of the simulation's overall goal.

Upon starting the program the subjects were faced with the task of using the arrow keys to effect physical movement throughout the simulated area. A series of statements were displayed on the screen that gave information to be used later in the solution of a question. This information may be stored in a "note book", a portion of computer memory that may be recalled at any time, by pressing the return key. When a question appeared the student must decide upon the appropriate method to arrive at the solution to the problem.

The first task was to select from the four mathematical operations of addition, subtraction, multiplication and division. After this selection was made the student had to find two appropriate number statements in the note book upon which the selected operation was performed. In the case of subtraction and division the order of the numeric statements was also critical to proper solution of the problem. The student can use the arrow keys on the computer to move back and forth through the list of

previously stored statements. A key could be pressed to select a highlighted numeric statement into the solution set or the numbers could be typed directly on the keyboard. After entering all of the necessary numeric statements the computer performed the indicated mathematical computation. After examining the results of the computation, the student was given the choice of continuing computation or having the computer program check the solution for accuracy.

The later stages of the computer programs simulated journey required the student to use a multi step and sometimes iterative process to solve the problems that were presented. An additional complication was introduced with the inclusion of "red herrings", that is statements which appeared as plausibly useful but having no bearing upon the proper solution of the problems. These problems added the intermediate solutions to the note book as they were derived so that they could be used in later steps. The operations needed for solution were also varied during the intermediate stages.

There are four programs in this series of problem solving exercises. Each is targeted at a different student grade level. The story line was changed for each program so that the characters and situations provided novel and unique situations in which to solve problems. Each of the simulations was designed to give students experience with number strands and operations in various applications and presented within a variety of strategies.

A short introduction to the computer and overview of the assignment was given to all subjects before the initial session. This overview included directions about computer operation, procedures required by the software and general information about mathematics problem solving as related to the computer software. Instruction in general mathematics problem solving techniques or cooperative learning skills was not given.

The study was conducted over a five day period in which students worked on a mathematics tutorial program in problem solving. The problem solving computer program is self paced and combines some instruction with problems that need to be solved before moving to the next lesson. Students were told that this is an enrichment unit that is expected to help them in further math classes.

Each pair of subjects worked on a computer in a space that was part of the library but visually separated from other students by five foot high barriers. Each treatment session was a class period of about 45 to 50 minutes.

Subject Grouping: The subjects in this study worked in groups of two students. The group size of two was used because it is the minimum in which to allow the opportunity for sharing to take place. Subjects were students in 7th grade math classes in an upper Midwest urban school district. They were randomly assigned to the treatment from their regular classrooms. A total of five pairs were used as subjects for this investigation.

Interaction Characteristics: Each subject worked in pairs to establish group problem solving characteristics. Subject performance in the group setting was taped using video equipment for analysis of both verbal and physical interactions. Coding of the data was completed by two independent coders. Descriptions of interaction categories as well as tallies of interactions will be provided in the results.

Mathematics Problem Solving Activity: The computer simulation presented a series of mathematics word problems that were to be completed in order to progress through the simulation toward an overall puzzle or riddle. Each of the computational

problems presented by the characters in the computer simulation were consistent with the general theme of the simulation and provided clues to the overall puzzle. The observation of the activities were coded into the broad categories of resources and heuristics and the further subdivided according to the follow subcategories:

Domain Knowledge

- Facts, definitions, etc.
- Algorithmic procedures
- Routine procedures
- Relevant competencies
- Knowledge about rules

Local Heuristics

- Review previous attempt
- Request help message
- Scroll through dialogue
- Paraphrase computer message
- Diagrams
- Pencil and paper
- Completed intermediate step
- Completed problem
- Select data for note pad

Cooperative Behavior

The subjects worked together in pairs on mathematics problems presented by the computer simulation. The simulation had an overall goal that became the focus for the pair so that both participants were mutually responsible for the achievement of the overall goal.

The basis for cooperation between individuals is their ability to communicate with each other in a manner that will allow each to contribute in a positive way toward a commonly defined goal. This communication is the transmission of both thoughts and feelings through words and actions that have a shared meaning. Each student must be able to send and receive accurately so that she may be able to "understand the other person's ideas, beliefs, feelings, opinions, reactions, needs, goals, interests, resources" and other things that may be related to the common objective (Johnson, 1975).

Cooperative Communications

- Complete idea
- Specific idea
- Congruent verbal and nonverbal
- Ask for feedback
- Give feedback
- Paraphrase partner message
- Negotiate meaning or understanding

This set of cooperative learning communication skills formed the basis for interaction and observation during the treatment. Subject interactions were coded on the basis of these six categories of communications.

Sequential Analysis

The distribution of mathematics problem solving characteristics exhibited by the subjects while engaged in the mathematics problem solving computer simulation were determined by data recorded on the mathematics problem solving portion of the interaction matrix. The record of the number and duration of mathematics problem solving interactions exhibited by the subjects was analyzed for both frequency and duration. These are presented as a percent of the mathematics problem solving activities.

The distribution of cooperative learning skills exhibited by the subjects while engaged in the mathematics problem solving computer simulation was determined by data recorded on the cooperative learning skills portion of the interaction matrix. The record of the number and duration of mathematics problem solving interactions exhibited by the subjects were analyzed for both frequency and duration. These are presented as both a percent of the activities and percent of the time spent in cooperative learning skills.

Two independent coders tallied results. Inter-coder agreement is $r = .83$ using Cohen's Kappa test. Within characteristics differences were examined through comparison of percentages for the mathematics problem solving and cooperative communications characteristics.

The flow of mathematics problem solving characteristics and cooperative communications skills were explored through sequential analysis techniques (Bakeman, 1986). Each possible pair of observed interactions are evaluated for probability of occurrence and statistical significance. Probability maps were developed for the mathematical problem solving characteristics and cooperative learning communications. Statistical significance was tested using parametric techniques (z-scores). An interaction matrix was used to represent sequential relationships between pairs of events. Frequency tables of subject by time period were used to examine changes in paired events over time.

Descriptions of subject actions while engaged in the computer mathematics problem solving activity during the initial treatment and paired condition are reported. Instances that indicate a difference in subject behavior that indicate a change in either mathematics problem solving characteristics or cooperative communications are illustrated.

Both the statistical analysis and descriptive narrative are used as a basis for forming conclusions.

Observational Frequencies

A tabulation of mathematics problem solving characteristics and communications characteristics was used as the basis of further analysis. Transcriptions of the student-to-student dialogue and nonverbal interaction are provided. Examples of data recording sheets are included in the appendix.

Transitional Probabilities

The sequential relationship between the occurrence of events was analyzed through probability of occurrence of those events. In the case of this study, the relationship of event A to event B is described by determining the probability event B following event A. This probability is then compared to the theoretical probability of this event sequence occurring at random. A statistical test was used to determine the statistical significance of the relationship between any two events.

Interaction Matrix

Interactions between subjects engaged in the mathematics problem solving computer activity was recorded on video tape during each session. These video tapes were then viewed by coders who have identified the time of onset and offset of each category of behavior given in the following observation matrix. Separate times are given for each cooperative learning communication skills and the mathematics problem solving skills. The cooperative communications skills and mathematics problem solving are anticipated to be nonexclusive behaviors so that the time of onset and offset of these overlapped each other. Coders were, however, required to differentiate between each of the behaviors exhibited by subjects within each of the cooperative and problem solving constructs.

Transitional Probability

The transitional probability schedule was derived from the actual data and is compared to the theoretical random probability of events. Each sequence of data has probability of occurrence based upon the actual observational data. This may be expressed as a calculated value between 0 and 1. Each sequence of data also has a theoretical probability based upon the likelihood of the events happening at random and has a calculated value between 0 and 1.

Each one tailed test with an $\alpha = .05$ is used for each possible sequence of transitional frequency in both the mathematics problem solving and cooperative learning strategies. A chart of transitional significance similar to the transitional frequency is used to display the statistical significance.

Observational vignettes

Narratives of interactions are used to provide insight into the nature of mathematics problem solving and cooperative learning strategies exhibited by the students. Mathematics problem solving within the context of the computer simulation is a complex activity. Descriptions of those interactions are essential to providing a full understanding of the problem solving activity. In addition, the character of cooperative interactions is as important as the frequency of interactions (Johnson, 1975). Vignettes that illustrate each of the problem solving and cooperative learning interactions exhibited by the subjects are included. These are used as a basis for conclusions as well as the statistical and frequency data.

RESULTS

These data were obtained by recording the interactions of five pairs of students. Each pair of subjects were randomly selected from a regular seventh grade mathematics class at an urban middle school in a medium sized upper Midwest metropolitan area. The subject pairs were assigned to a temporary laboratory setting within the instructional materials center of the middle school. The computer and video/audio taping equipment were set up each day before the arrival of the students and removed from the area after the students had completed each session. Each subject pair participated in five computer mathematics problem solving simulation sessions of about forty-five minutes in duration. Pencil and paper was provided for the subjects during each session.

The flow of mathematics problem solving characteristics and cooperative learning skills were explored through sequential analysis techniques (Bakeman, 1986). Each possible pair of observational events were evaluated for probability of occurrence and statistical significance. Statistical significance is tested using parametric techniques (z-scores). An interaction matrix is used to graphically represent statistical significance of event pairs with only the event pairs with a $p < .05$ identified in table 1.

The likelihood of a pair of sequential events occurring in relation to all other event pairs was determined by using the Bakeman and Gottman statistical analysis technique previously described in the paper. These are given as onset, the first of two events, and the offset, the second of the events. Each onset may originate with either the first or second of the paired subjects with the offset the event undertaken by the other of the pair. Only the transitional events that occur between the subjects are tallied. Event sequences occurring within the same subject were not tallied during this analysis. The pair of events, number of occurrences during the sessions and the tally of occurrences by the first and second halves and first and second subject are listed.

An example of the dialogue between subjects for a sample of the event pairs is given in order to illustrate the way in which the subject pairs interacted while proceeding through the computer mathematics problem solving simulation. Descriptions of subject actions while engaged in the computer mathematics problem solving activity during the initial treatment and paired condition are also reported. Instances that indicate a difference in subject behavior that indicate a change in either mathematics problem solving characteristics or cooperative communications are identified. Interactions between the subjects and the media of instruction, the computer, will also be examined for possible influence on the outcome of the activity.

A tabulation of mathematics problem solving characteristics and communications characteristics is used as the basis of further analysis. Transcriptions of the student-to-student dialogue and nonverbal interaction that illustrate the significant transitional events are provided.

The following discussion is an example of the interaction between subject 1 and subject 2 for subject pair B. The transitional cooperative communications characteristic onset event "ask for feedback" and offset event "give feedback" were found to be significantly related. The frequency for the subject pair B over time is given in table 1.

Onset: Ask for Feedback
 Offset: Give Feedback

Subject
 s1 s2

Table 1. Frequency of transitional events for subject pair B.

Period	Subject	
	s1	s2
1st	6	13
2nd	4	4

In the first half of the mathematics problem solving simulation, subject 1 asked for feedback and subject 2 gave feedback six times. Subject 2 was responsible for the onset and subject 1 the offset of this event sequence 13 times. In the second half of the time spent by pair B, both subject 1 and subject 2 were responsible for 4 onset and offset actions. It appears that the actions of subject 1 and subject 2 were very dissimilar during the first half of their work with the mathematics problem solving simulation. The frequency of onset and offset of asking for and giving feedback became the same during the second half of the interactions.

In regard to asking and giving feedback, subject 2 initiated a question and subject 1 responded more than twice as often during the first half of their interactions. It appears that subject 1 had a greater understanding of the procedures necessary to move towards solution of the mathematics problems and was willing to share that understanding. During the second half of the interactions, both subject 2 as well as subject 1 increased their expertise in arriving at the solutions of the problem which resulted in the reduction of the incidents of information exchange between the pair of subjects.

The following sample event stream illustrated a typical interchange in the first and second halves of the computer activity. The transcription of dialogue between subject 1 and subject 2 is given in quotations.

- | | | | |
|----|------------------|----|--|
| 1 | | s2 | scrolls through the computer simulation |
| 2 | | s2 | selects information for the note pad |
| 3 | Give Feedback | s1 | "Plus" |
| 4 | Give Feedback | s2 | selects + 29 + 15 (=44) |
| 5 | Give Feedback | s1 | "Not that far" |
| 6 | Give Feedback | s2 | "That's right" |
| 7 | Give Feedback | s1 | "Uhhh" (no) |
| 8 | Give Feedback | s2 | "Yes it is" |
| 9 | Give Feedback | s1 | "Uhhh; you're supposed to add this up" (points) "and subtract that" (points) |
| 10 | Give Feedback | s2 | "Ah, ok, ya" selects + "go back up to 29+24" |
| 11 | | s1 | "There that's the answer" |
| 12 | Ask for Feedback | s2 | "What about 15?" |
| 13 | Give Feedback | s1 | "Doesn't matter....." |
| 14 | Ask for Feedback | s2 | "And now this?" (refers to answer) |
| 15 | Give Feedback | s1 | "No, no, no, now you subtract" |
| 16 | Ask for Feedback | s2 | selects - "53" |
| 17 | Give Feedback | s1 | "Ya go to 53" |
| 18 | | s2 | moves to 53 |

19		s1	"Up to 15"
20		s2	moves to 15
21	Give Feedback	s1	"Return" uses keyboard to move to answer
22		s2	selects answer
23	Give Feedback	s2	"Ya, got it right"

Second half of activity.

24		s1	scrolls through the character Bert
25		s1	selects information for the note pad
26	Give Feedback	s2	"Go to the note pad"
27		s1	"No problem, times that" points selects $x 27 - x 6 = 16....$
28	Give Feedback	s1	"Now subtract"
29		s1	select $16.-2 +$
30	Ask for Feedback	s1	"What did I do wrong?"
31	Give Feedback	s2	"I think it...." points to screen
32	Give Feedback	s1	"Oh, no times - subtract"
33		s1	subtract $46 - 16 = 30,000$
34		s1	selects answer
35		s2	scrolls through the character
36		s2	selects information for the note pad
37		s1	selects information for the note pad
38		s2	selects information for the note pad
39	Ask for Feedback	s1	"What did you do so fast for?"
40	Give Feedback	s1	"Oh - add"
41		s2	chooses addition operation
42	Give Feedback	s1	"That plus that"
43		s2	selects the addends for computation
44	Give Feedback	s1	"Now divide that by 12"
45		s2	completes the division and then discusses the cursor location
46		s2	selects answer

These interchanges are examples of a shift of initiation in making sense of the current mathematics problem from one member of the subject pair to the other at two different times. In lines 3 through 9, subject 1 and subject 2 are engaging in a dialogue that does not require an explicit question in order to give feedback about the problem. Each in turn assists in making sense out of the mathematics problem and suggesting ways in which to proceed. The dialogue in lines 11 through 18 show the subjects engaging in an interchange in which subject 2 is asking subject 1 specific questions and subject 1 responds to those questions. In lines 26 through 34, during the second half of the coded events, subject 1 is requesting assistance from subject 2 and then receiving a response. The dialogue indicates that the individuals in subject pair B engaged in a cooperative communications about the current mathematics problem presented by the computer simulation in order to find the solution to the problem and that the responsibility for initiating inquires and providing responses may be dynamically adjusted during the problem solving episode.

Each of the subject pairs engaged in a number of event pairs that proved to be significant. Figure 1 gives a summary of the significant event pairs and graphically represents the categories into which those events fall. There are a total of eleven event pairs that are statistically significant for the subject pairs in this study. The following is a summary of those events.

Asking and giving feedback was a significant transitional event pair for subject pairs A, B and C. This indicates that part of the progress to solution to the mathematics problems may be the result of cooperative communications for these pairs but did not stand out for the other pairs.

Giving feedback along with the completion of the problem was a significant transitional event pair for the subjects pairs A, B and D. This may indicate that for these subjects, the giving of information is immediately linked to the solution of the mathematics problems.

The event of algorithmic procedure paired with another algorithmic appeared significant for subject pairs A, B and E. The determination of an appropriate algorithmic procedure to solve a problem seems to be an activity that requires interaction between both subjects in a pair to reach agreement.

An algorithmic procedure paired with a routine procedure was significant for subject pair E. All other subject pairs were able to move through the simulation without a significant number of algorithmic to routine procedures occurring. This may indicate a unique problem solving style demonstrated by the subject pair.

Routine procedure to giving feedback were significant transitional events for subject pairs B and D. For these subjects the execution of a routine procedure was followed by an unsolicited comment on the part of the other.

Scrolling through the computer simulation dialogue followed by selecting data for the note pad was a significant transitional event for subject pairs C, D and E. This appears to indicate that for these subjects the selecting of data after scrolling form an important sequence.

Paraphrasing the computer message with communicating a specific idea was a significant event pair for subject pair C, D and E. This may indicate that a characteristic of the problem solving style for this pair was to read back the computer dialogue and then articulate a idea that was related to the dialogue.

Completing a problem with scrolling through the computer dialogue was a significant event pair for subject pairs B, C, D and E. This may indicate that the subject pair A has a unique characteristic of the problem solving style that was not in evidence for any other pair making this an unimportant event for this pair. All other pairs demonstrated a significant engagement in the event pair.

Selecting data for the note pad followed by the articulation of a specific idea was a significant event pair for subject pairs A and D. It appears that for these subjects, after one of the pair entered data the other one of the pair would make a comment about that activity.

Selecting data for the note pad followed by scrolling through the computer dialogue was a significant transitional event for subject pairs C, D and E. For these two subject pairs, continued scrolling through the dialogue after entry of data formed an important

characteristic of their problem solving strategy.

In summary, each of the subject pairs are somewhat unique in the way that they solve the mathematics problems presented by the computer simulation, however, they also show some common characteristics. Only the statistically significant event sequence of algorithmic followed by routine procedure was exhibited by only one subject pair. All other event pairs were commonly shared by one or more of the subject pairs.

Relative Distribution Of Events

Although the general pattern of problem solving seems to be fairly consistent, each of the subject pairs has developed different patterns of interactions in order to complete the mathematics problem solving activities in the computer simulation. Those patterns appear to be distributed in ways that reflect both their level of expertise in problem solving and their ability to work together toward common aims. Figure 2 illustrates both the similarities and differences in the way that the subject pairs approach the problem solving activity.

The general distribution for each characteristic category is similar for all subject pairs. This indicates that even though there are differences in the way that each pair of subjects goes about the task of solving the mathematics problems, there are common patterns of activity that lead toward that goal. It should also be noted that some of the traditional strategies for problem solving either were not used or made explicit during the activity.

General heuristics identified by Schoenfeld (1989) were almost entirely absent as part of the subject's problem solving repertoire while the domain knowledge of mathematics was very apparent. This may indicate that this computer software does not provide an environment in which these skills are important. Local heuristics that were a necessary component in the solution of the mathematics problems presented by the computer were both evident and appeared in greater number.

In summary, the relative distribution of mathematics problem solving activities indicates that subjects did not exhibit all of the problem solving characteristics under investigation in this study. Second, the overall distribution of events forms a consistent pattern for all of the subject pairs. Third, there are marked differences within the subject pairs in the relative distribution of event types when compared with other subjects.

The distribution in domain knowledge is fairly consistent in that all of the subject pairs seem to spend the most time with algorithmic procedures and routine procedures. Little time was devoted to the determination of facts or definitions. Instances of explicit action indicating relevant competence and knowledge about rules were relatively few.

Most of the domain knowledge category of activity is devoted to determination of the appropriate algorithmic procedures and routine procedures. There are differences in the proportion of algorithmic procedures and routine procedures between subject pairs. These differences are most apparent between subject pair A and subject pair C. Subject pair A engaged almost equally in determining algorithmic procedures and routine procedures. This ratio seems to indicate that for this subject pair it took no more time to determine the way in which a problem was to be solved, algorithmic procedures, as it was to carry out the solution of the problem, routine procedure. Subject pair

C, however, devoted a large proportion of their actions towards determining the appropriate way to solve the problem, algorithmic procedure, than to actually solving the problem, routine procedure. Other subject pairs in the study fell between these two extremes. They did, however, expend a greater effort in determining the appropriate algorithmic procedure than in demonstrating the routine procedure.

DISCUSSION

This study indicates the interactions of subjects who are engaged in the solution of mathematics problems presented to them through a computer assisted instruction simulation. Subjects in this study modified their interactions in ways that appear to ensure success in mathematics problem solving. Event sequences were modified between the first and second half of the activity that reflect the growth of understanding in both the local problem solving heuristics and the domain knowledge characteristics. Subjects in subject pair A at times would refer to the way in which the current problem was similar to an earlier problem. This notion of change and adaptation could also be seen in the way in which the cooperative communications event sequences were modified. Three of the pairs experienced a decrease in the total number and difference between the number of times that a sequence of questions and answers were necessary during their progress through a mathematics problem.

Subjects in the study maintained progress in solving the mathematics problems without direct instruction from the researcher or teacher. Some time was necessary in which the subjects struggled with the problems in order to construct an understanding of the way that the problem could be solved. The number of incidents in which it was necessary to construct understanding decreased between the first and second time periods in the study. Direct instruction was not necessary for successful completion of the simulation. This indicates that the subject pairs were able to construct their own knowledge and understanding of mathematics problem solving during the computer activity. This may be especially helpful for teachers who are would like to provide their students with mathematics problem solving activities that do not require constant monitoring or progress.

Subjects displayed differences in distribution of problem solving and cooperative communications characteristics. Although a great deal of similarity in the problem solving interaction events was present there was also a variation between the subject pairs. Some of the subjects spent more time on discussion of routine than algorithmic procedures than others. Some pairs devoted a larger amount of effort to master the intricacies of the computer software.

Individual differences existed within each of the pairs. Some members of the subject pairs contributed a greater number of events than others. This difference appears to be tied to the amount of knowledge of mathematics procedures and willingness to give information. The dominate individual, however, did assist the other subject in the pair develop strategies in problem solving that were reflected in a change in event distribution between the first and second halves of the coded events.

Consultation with each other was an accepted and often used strategy for determining appropriate responses to problems. Subjects were readily willing to engage in expressing their knowledge and understanding about the procedure for

mathematics problem solving. These interchanges were in keeping with the model of cooperative communications developed by Johnson and Johnson. It appears that subjects who were not particularly adept at cooperative communications, as shown by the number of coded events, became more like their partner in cooperative characteristics during the second half of the computer activity. This may indicate that the computer simulation could also have value in giving students the opportunity to learn and practice cooperative skills that could be used in other classroom activities.

Limitations

The sample size in this study makes general inferences about the distribution of problem solving characteristics problematic. Although the subjects were mixed race and gender, it would be presumptuous to assume that the observations made would necessarily reoccur in other students. Additional studies with a larger number of students combined with inferential analysis techniques would be helpful in drawing conclusions about the possible benefits for larger groups of classroom size.

No attempt has been made at a quantitative analysis of differences between subject pairs or within pairs. Differences in learning styles, learner control, ethnic or gender differences, attitudes and aptitude have not as yet been explored. There may be value in exploring these other areas to round out the picture of mathematics problem solving within the limitations of this computer simulation.

Summary

Mathematics instruction is in the processes of evolution towards a greater emphasis on problem solving with the Standards as one of the catalysts for that change. Cooperative learning methods combined with the appropriate use of computer technology may be able to play a part in this change by providing educators with another way in which to organize instruction and present problem solving challenges. This study indicates that for this limited case mathematics problem solving skills along with cooperative communications can be a positive supplement to more traditional instruction. Additional work in this area is indicated in order to assess the extent and magnitude of the learning taking place during the completion of the computer simulation.

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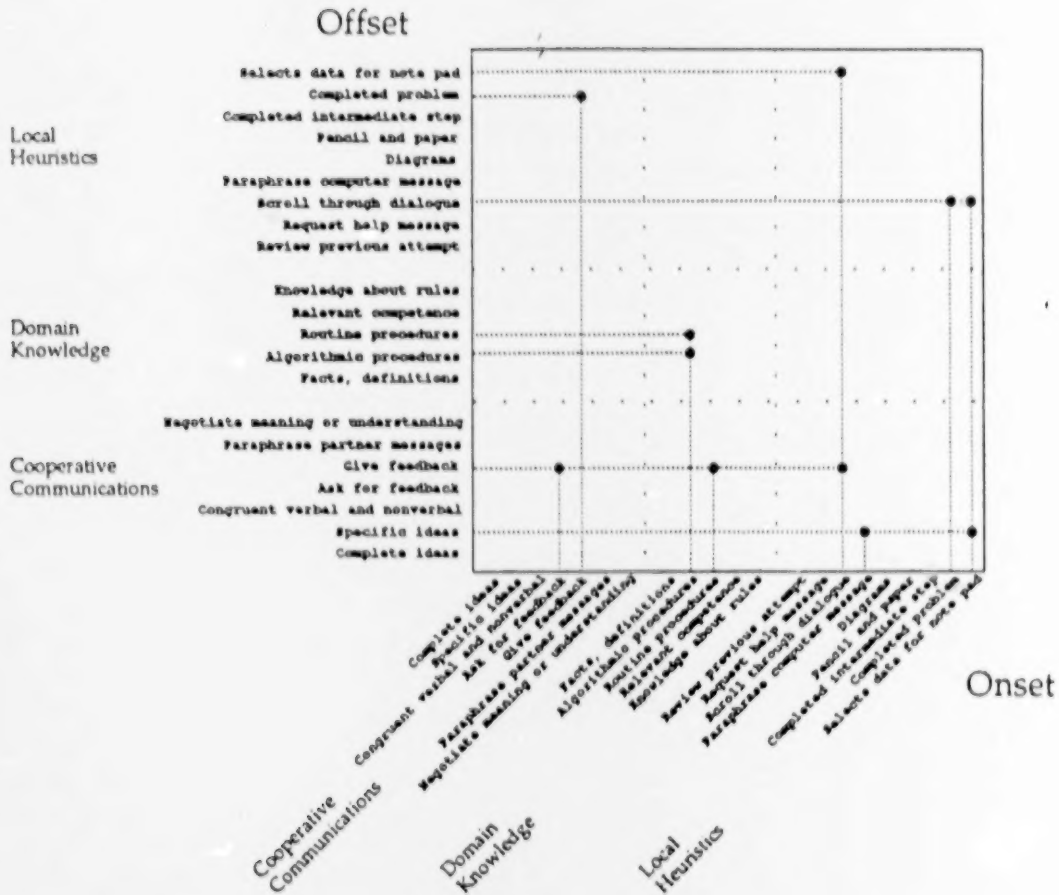
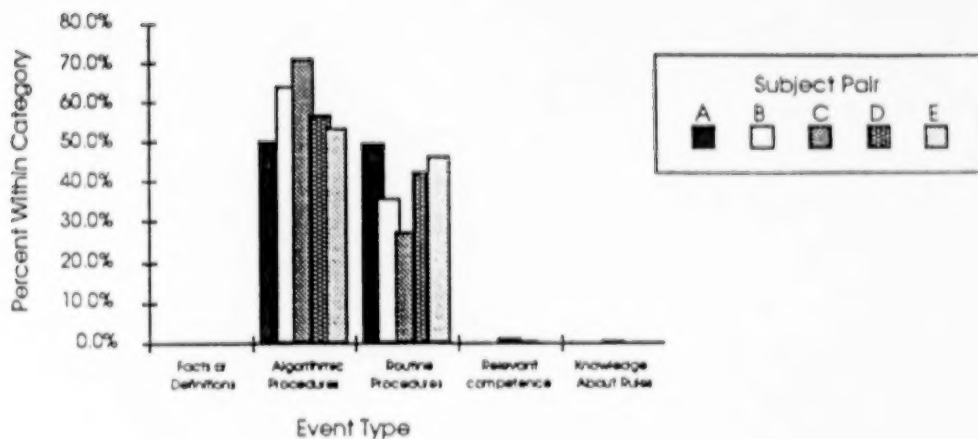


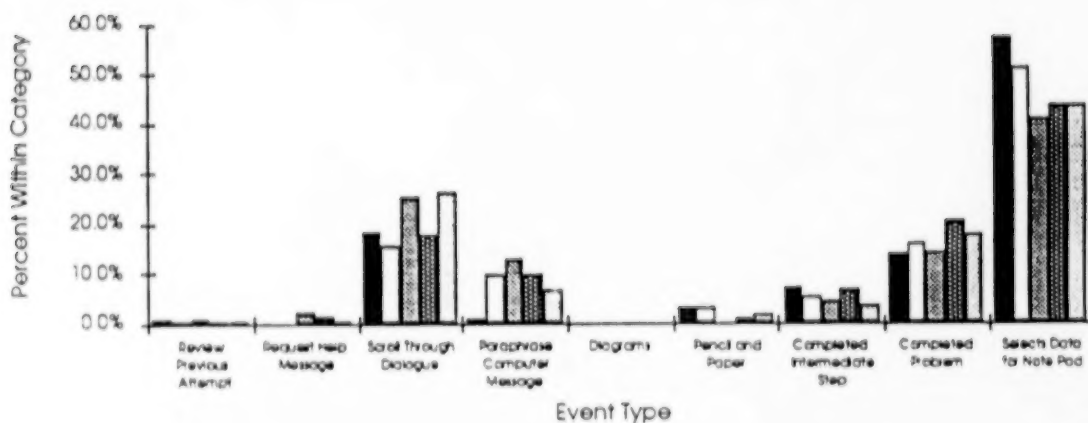
Figure 1. Transitional event onset to offset sequences for all subject pairs with $p < .05$ by event characteristic.

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Domain Knowledge



Local Heuristics



Cooperative Communications

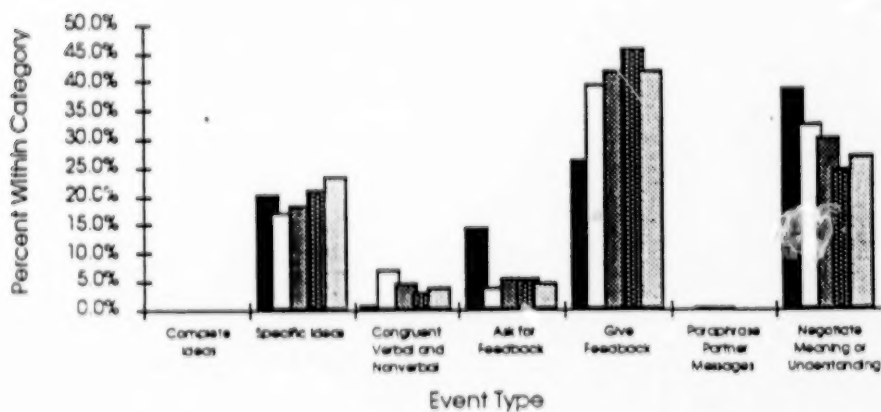


Figure 2. Relative distribution of events within mathematics problem solving and cooperative communications categories for all subject pairs in each event type.